Title Of Unit: Introducing Logarithms **Subject/Course/Grade: Algebra 2/11th**

Dates of Unit: 4 Days **Designer: Marissa Misura**

Stage 1- Desired Results

Standards:

- CCSS.MATH.CONTENT.HSF.BF.B.5

Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

- CCSS.MATH.CONTENT.HSF.LE.A.4

For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Essential Questions

- What is a logarithm?
- How do I evaluate a logarithm?
- Why is a logarithm useful?

Students will be able to …

- Evaluate a logarithm without a calculator
- Approximate a logarithm without a calculator
- Use properties of logarithms to evaluate unknown logarithms
- Use a calculator to evaluate logarithms
- Use a graphical model to approximate real world data

Stage 2- Assessment Evidence

Monitoring and Feedback:

- Students will work in small groups
- Students will discuss small group work as an entire class
- Students will practice on their own in class with teacher giving feedback
- Students will practice at home, with answers given the next day

Other Evidence:

- Call and response
- Number talks
- White board review
- Daily Homework assignments
- Death of a Salesman activity

Sample questions students should be able to answer at the end of unit:

- Evaluate $\log_3 81$, $\log_{1/2} 32$, log1000, $\log_7 7$
- Expand: log $(3x^2/y)$
- Condense: ln 4 + ln x - 3ln y
- Use Newton's Law of Cooling to figure out how much time has passed when heating a cup of water for tea and then letting it sit out to cool down. T = current temperature of the heated water (100), Rt is the constant room temperature (72), and replace 98.6 with the original temperature of the heated water (185).

Stage 3- Learning Plan

Learning Activities:

Where students are coming from: We have just finished a mini unit on function operations and inverse functions. Next we moved onto exponential functions, evaluating, graphing, modeling, etc. And finally students will be introduced to logarithms. Having the knowledge of inverse functions is important and exponential functions is important as understanding both help to better understand where a logarithm comes from. It is important to help the students make these connections so that they can better understand where logarithms come from.

I would suggest a week to two weeks before you begin this lesson have a warm up question each day along the lines of '2 to what power is 8?' and any variation of that. This will get students thinking about the question that a logarithm is asking so when we begin to evaluate logarithms it isn't completely foreign.

Day 1: Introducing Logarithms as an inverse of Exponents

Remind students about exponents and the question you have been asking for a few weeks… 'x to what power is y'? This is a LONG way to ask a pretty simple question so mathematicians made a shortcut for that question - LOGARITHMS! Today we are going to explore how to evaluate logarithms some more.

Using the lab: Number Sense: Properties of Logs have students work in groups of 3-4 to complete each task on the FRONT side only (the back side will be completed tomorrow) using only their brains! It will be important to constantly remind students that seeing '$\log_8 2$' is just asking the question '8 to what power is 2?' so that they do not need to use a calculator to figure out any logarithms as all of these evaluate to rational numbers.

As students work, walk around to groups to make sure they are getting through the lab together as a group. Listen for students speaking about logarithms correctly for example: $\log_4 64$ we say - 'log base 4 of 64 is…' or listen for students to be saying '4 to what power is 64?' You may hear answers like 16 - remind them we are not multiplying to get the number we are counting multiple 4's - so we need 3 total 4's to multiply to get a 64. Students may struggle with fractional bases or when taking the logarithm of a fractional number at this point - remind them that negative exponents will allow us to make fractions. Students may also struggle when the base is larger than the number you are taking a logarithm of - remind them that fractional exponents are like roots and roots will make numbers smaller!

After 15-20 minutes bring students together as a class and discuss their answers. Have different groups answer each question and when providing extra examples speak with multiple other groups to get a lot of different examples for each summarizing question. Double check with other groups that examples work out correctly and if they do not what adjustments can be made to make it correct. Come to a consensus as a class when 'describing solutions of a=b, a>b, a<b, etc'...

When a = b your answer will always be 1

When a>b your answer will always be fractionally between 0 and 1

When a<b your answer will always be a whole number

When a is fractional your answer will always be a negative number

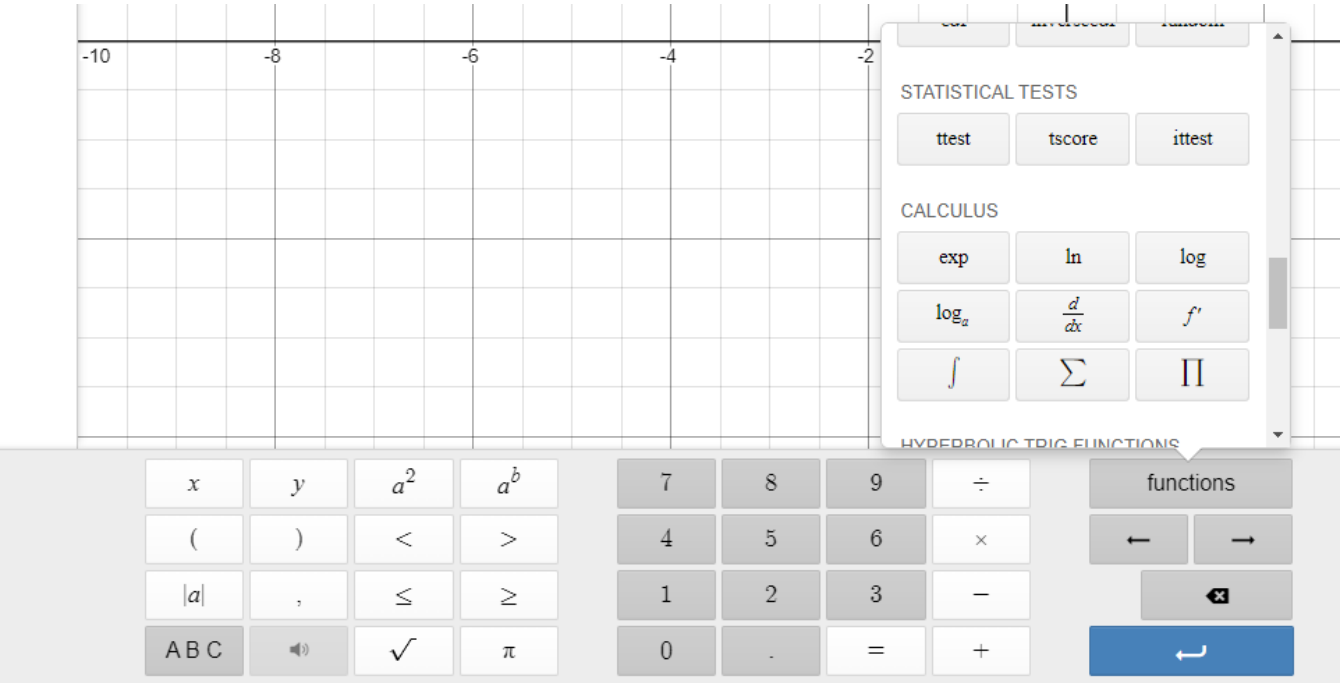
Talk about base 10 being the common base (so if you do not see a base, it is 10) and when the base of your exponent is 'e' we use a natural logarithm (ln) instead - but it means the SAME thing!

For additional practice present the following slide show and have students evaluate logarithms in their head: Logarithm Extra Practice

For Homework: Evaluating Logs

Day 2:Introducing Properties of Logarithms
Today we want to look more at how to add, subtract, multiply, and divide logarithms!

Again in groups of 3-4 have students work through the lab: Math Lab:Properties of Logs I would let students know it may SEEM like you cannot evaluate these logarithms in your head, but there is a pattern to notice. However, if the entire group is struggling to see the pattern after several minutes, I would direct them to use desmos.com to evaluate some of the logarithms and then look again for a pattern. Using desmos.com on either your chromebook or phone go to desmos.com>graphing calculator. I would show students on an overhead projector where the logarithmic function button is, and how to type in logarithms a few times to practice before having them begin their lab for the day. I would then push students to only use this for the first few problems in each row and then try to look again for the pattern and then STOP using a calculator.



Give students 15-20 minutes to complete the entire worksheet front and back. Then come together as a class to discuss their findings about the Product Property, Quotient Property, and Power Property. As you work through the worksheet highlight #7, 9, and 11 and add the words (and have students add the words) Product Property, Quotient Property, and Power Property. Connect these to the rules that they learned during the exponent unit as well, such as:

Do you remember what happens when we are multiplying exponents with the same base? (we add the exponents and keep only one base) This is just like that, but opposite! When we are adding logarithms, we multiply what is inside them and keep only one logarithm.

Do you remember what happens when we are dividing exponents with the same base? (we subtract exponents and keep only one base) This is just like that but opposite. When subtracting logarithms, we divide what is inside and keep only one logarithm.

After a whole class discussion students should then go back to yesterday's lab and complete the back using their new properties. Come together as a class one more time to go over their findings after completing yesterday's lab.

Students may be tempted to actually multiply or divide these numbers - tell them that it seems like it wouldn't be much fun because of how large the numbers are. Instead, use our properties (Product and Quotient) to break up multiplication to addition and division to subtraction.

Also remind students the question that a logarithm is asking - '2 to what power is 16' and '2 to what power is 256'? Now since these are being multiplied what do I do with those two results. Remind students multiplication leads to addition in logarithms because they are inverses of exponents.

Homework: Practice Properties of Logarithms tell students they will not be evaluating anything, simply using their new properties to expand or condense.

Day 3: Review Evaluating Logarithms and Properties of Logarithms
Each student will be given a white board, dry erase marker (my white boards are just sheet protectors with white card stock inside of them), and a napkin to erase answers. Using the slide show from before (Logarithms Practice) go over the directions for 'how to play' with students and then begin. Give students plenty of time to think about and write down their answers. Encourage students who get it correct early to help their neighbors. After each question either simplify the question yourself and explain the process, or have a student volunteer come up to show how to simplify each question. If you want to make it more of a game, have students make tally marks for each question they get correct and then at the end see who got the most correct.

Homework: Practice Expand and Condense Logarithms

Day 4: Real Life Applications of Logarithms
Using our Logarithm Extra Practice slideshow introduce students to a few of the ways logarithms are used including the one we will use today, Newton's Law of Cooling!

Have students again in groups of 3-4 begin working on Math Journal: Death of a Salesman. I would recommend reading the introduction to all students, pointing out specifically what each variable in the formula represents and that you have information to read in the journal and ask where the 70 in the formula comes from? Where does the 98.6 come from? Don't forget to multiply by the -10 after you take the natural logarithm!

Students may use desmos.com as a graphing calculator - remind them to use the NATURAL log button and where to locate that. They do not really need to graph (they may if they want to see what a logarithmic function looks like as we will get into graphing in the next unit) however they cannot evaluate these logarithms without a calculator.

Give students 25-30 minutes to complete the activity and then let groups argue out who they think killed the salesman!

As students are working monitor that they are typing the natural logarithm into desmos correctly. You can even ask, does this answer make sense (the base of a natural log is e approximately 3 and we are taking

a logarithm of approximately 15, since a<b my answer should be bigger than 1 maybe around 2 ish because 3 to the 2nd power is 9, and 3 to the 3rd power is too much at 27)

Students may struggle to subtract time because it is base 60, advise them that they can count as well.

Remind students what an average is.