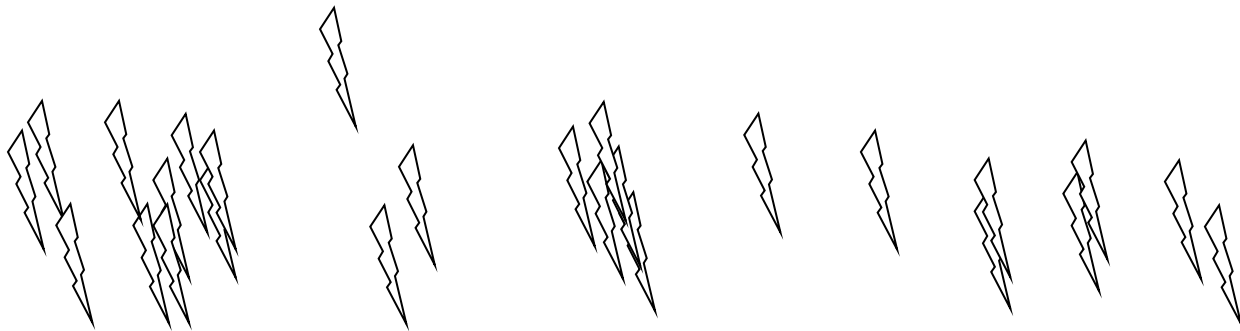


Counting Items Quickly



Sucks Right? However...



How many DIFFERENT lunches??

Cafeteria Menu

Sandwich:

Hamburger
Cheeseburger
Veggieburger
PB&J

Side:

Fries
Beans
Corn

Dessert

Apple Crisp
Banana
Pudding
Chocolate Cake

- **1. A quick question:** A totally made up fact: there were only four different types of dinosaurs: the Iguanodon, the Juravenator, the Allosaurus, and the Gigantosaurus. Each dinosaur came in one of three colors: red, purple, and blue. The Museum of Natural History wants to have a model of each different-looking dinosaur. How many different models does the museum need to create? And more importantly: *somehow convince me that your response is correct* .
- **2. Another quick question:** There is one more additional piece of made-up fact for you to consider. Each dinosaur had one, two, three, four, or even five claws. With this additional piece of information, how many different models does the museum need to create? And more importantly: *somehow convince me that your response is correct* .

How many more license plates were available in 2004 vs 1912?



Fundamental Counting Principle

- If two, three, or more events can occur m , n , etc number of ways, then the number of ways both or all can occur is: $m \cdot n \cdot etc$

Practice Fundamental Counting

- 1) You are buying pizza, you have a choice of 3 crusts, 4 cheeses, 5 meat toppings, and 8 vegetable toppings. How many different pizzas with one crust, one cheese, one meat and one vegetable can you choose?
- 2) FHS has telephone numbers that all begin with 591 followed by 4 digits. How many different phone numbers are possible? (a) if digits can be repeated and (b) if digits cannot be repeated?

Combinations and Permutations

$${}^n C_r = \frac{n!}{(n-r)! \cdot r!}$$

$${}^n P_r = \frac{n!}{(n-r)!}$$

n → Total number of objects

r → Number of objects you are choosing

C = Combinations

P = Permutations

! = excitement! just kidding... FACTORIAL

What's the difference?

- In English we use the word 'combination' loosely, without thinking if the order of things is important. For example:
 - My fruit salad is a combination of apples, grapes and bananas (here the order doesn't matter, it's all delicious)
 - The combination to my locker is 6-31-14 (here we do care about the order, otherwise your locker won't open)
- In math, we must be more precise
 - If the order **doesn't** matter it is a combination, C, fruit salad
 - If the order DOES matter it is a permutation, P, locker combo

So does order matter? Or nah?

Order DOES NOT matter

- A chemistry class is divided into 12 groups, each group submits one drawing of the molecular structure of water. The teacher will pick four of the drawings to show.

- Combinations

Order DOES matter

- You will draw winners from a total of 25 tickets. First place will win \$200, second place will win \$100, and third will win \$50.

- Permutations

Permutations - Examples

Playoffs

- 1) Eight teams are competing in a baseball playoff
 - a) In how many ways can the baseball teams finish the competition?
 - b) In how many different ways can 3 of the baseball teams finish first, second and third?

Homework

- 1) You have 6 homework assignments to complete over the weekend. However, you only have time to complete 4 of them on Saturday. In how many orders can you complete 4 of the assignments?

Permutations – with repetition

- Find the number of distinguishable permutations of the letters in the following...
 - a) Tomorrow
 - b) Yesterday
 - c) Even
 - d) California
- If there is repetition you must divide these out!
- Permutations of n objects where something is repeated s times, another s_2 times, etc... is:
$$\frac{n!}{s! s_2! \cdots s_k!}$$

More Permutations

1) How many different 7 digit telephone numbers are possible if all digits can be repeated?

2) If you have 9 chores to do, in how many orders can you do 5 of them?

Combinations

1) You are picking 7 books from a stack of 32. if the order of the books you choose is not important, how many different 7 book groups are possible?

2) There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?

Permutation or Combination and solve.

P or C?

- 1) How many different teams of 11 players can be chosen from a soccer team of 16?
- 2) Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?
- 3) A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?

Solve.

- 1) combinations, 4368
- 2) Combinations, 21
- 3) Combinations, 70

Permutation or Combination and solve.

P or C?

- 1) You and your friends are picking up videos at the video store. You have selected 7 videos but will only have time to watch 3 together.
- 2) A car door lock has a five button keypad. Each button has two numerals. The entry code 21914 uses the same button sequence as the code 11023. how many different five button codes are there, you can use a button more than once.

Solve.

- 1) combinations, 35
- 2) permutations, 3125

More Examples!

Permutations

- How many permutations of 3 different digits are there, chosen from the ten digits 0 to 9 inclusive?
- 720

Combinations

- An old website requires a four-character password consisting of three numbers and one letter. A new website requires a six-character password consisting of three numbers and three letters. How many more passwords can be made for the new website?
- 17,550,000

Permutations

- How many permutations of 4 different letters are there, chosen from the twenty-six letters of the alphabet?
- 358,800

Combinations

- How many different committees of 5 people can be chosen from 10 people?
- 252 committees

Permutations

- In how many ways can you arrange nine CDs one after another on a shelf?
- 362,880

Fundamental Counting

- A special type of password consists of four different letters of the alphabet, where each letter is used only once. How many different possible passwords are there?
- 358,800

Permutations

- You have 20 songs on your iPhone. You have time to listen to four of the songs. In how many different ways can you select four songs if order does not matter?

Fundamental Counting

- A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?
- 676,000

Permutations

- An encyclopedia has 8 volumes. In how many ways can the eight volumes be placed on the shelf?
- 40,320

Probability and Combinations

Playing Cards Example...

- What is the theoretical probability of being dealt exactly two 7's in a 5-card hand from a standard 52 card deck?
- 4%
- What is the theoretical probability of being dealt all four 7's in a 5-card hand?
- 0.00185%

Games

A group of 30 students from your schools is part of the audience for a TV game show. The total number of people in the audience is 150. What is the theoretical probability of 3 students from your school being selected as contestants out of the 9 possible contestant spots?

17.9%

More Examples!

Political Campaign...

A politician is about to give a campaign speech and is holding a stack of eight cue cards, of which the first 3 are the most important. Just before the speech, he drops all of the cards, and picks them up in a random order. What is the probability that cards 1, 2, and 3 are still in the order on the top of the stack?

1 way this happens.

How many possible ways?

Does order matter?

.003 or .3%

Gardening

A gardener has 11 identical looking tulip bulbs of which 6 will produce yellow tulips and 5 will become pink. She randomly selects and plants 6 of them and then gives the rest away. When the flowers start to bloom, what is the probability that all of them are yellow?

How many are yellow and how many are we selecting? Does order matter?

How many total ways can we select 6 tulips?

.2%

Guessing on your Test

A test consists of seven true/false questions. A student who forgot to study guesses randomly on every question. What is the probability that the student answers example four questions correctly?

How many outcomes do we want? Does order matter?

What is the total number of outcomes?

27.3%

Sports

A basketball player has a 50% chance of making each free throw. What is the probability that the player makes exactly three out of nine free throws?

How many outcomes do we want? Does order matter?

What are the total number of outcomes?

16.4%

More Sports

A basketball player has a 50% chance of making each free throw. What is the probability that the player makes at most eight out of ten free throws? 98.9%

Coins

A fair coin is flipped fifteen times. What is the probability of the coin landing tails up at most thirteen times? 99.9%

More Politics

A meeting takes place between a diplomat and seventeen government officials. However, ten of the officials are actually spies! If the diplomat gives secret information to eight of the attendees, what is the probability that the diplomat gave secret information to exactly five spies?

36.3%

Homework!

Katelynn is carrying six pages of math homework and four pages of English homework. A gust of wind blows the pages out of her hands and she is only able to recover seven random pages. What is the probability that she recovers at least 5 pages of her math homework?

33.3%

Marbles...

A jar contains 30 red marbles, 50 blue marbles, and 20 white marbles. You pick one marble from the jar at random.

- | | | |
|----|----------------|-----|
| 1) | P(red) | 30% |
| 2) | P(blue) | 50% |
| 3) | P(not white) | 80% |
| 4) | P(red or blue) | 80% |

Blocks...

A bag contains 36 red blocks, 48 green blocks, 22 yellow blocks and 19 purple blocks. You pick 1 block from the bag at random.

1)	P(green)	38.4%
2)	P(purple)	15.2%
3)	P(not yellow)	82.4%
4)	P(green or yellow)	56%
5)	P(yellow or not green)	61.6%
6)	P(purple or not red)	71.2%

Probability of Multiple Events

When to Add and When to Multiply!

You Add!

Dependent or Independent?

You Multiply!

- | | |
|---|----------------|
| 1) Roll a numbered die. Then spin a spinner | 1) Independent |
| 2) Pick one flash card, then another from the remaining. | 2) Dependent |
| 3) Select a coin at random from your purse, put it back and pick another | 3) Independent |
| 4) Pick a month, pick a number 1-30 | 4) Independent |
| 5) A month is selected, a day of that month is selected | 5) Dependent |
| 6) A letter of the alphabet is selected, one remaining letter is selected | 6) Dependent |
| 7) The color of a car is selected, the type of transmission is selected | 7) independent |

Using Combinations...

The local movie rental store is having a special on new releases. The new releases consist of 12 comedies, 8 action, 7 drama, 5 suspense, and 9 family movies.

- a) You want exactly 2 comedies and 3 family movies. How many different movie combinations can you rent?
- b) You can afford at most 2 movies. How many movie combinations can you rent?

Mutually Exclusive or No?

- | | |
|---|-----------|
| 1) The sum is a prime number; the sum is less than 4 | 1) no, 2 |
| 2) The numbers are equal, the sum is odd | 2) Yes |
| 3) The product is greater than 20, the product is a multiple of 3 | 3) No, 24 |

Examples!

1) Suppose you have five books in your book bag. Three are novels, one is a biography and one is a poetry book. Today you grab one book out of your bag without looking, and return it later. Tomorrow you do the same thing. What is the probability that you grab a novel both days?

2) About 30% of the U.S. population is under 20 years old. About 17% of the population is over 60. what is the probability that a person chosen at random is under 20 or over 60?

47%

Standard Die

- | | |
|-----------------------------|------------------|
| 1) P(3 or odd) | 1) $\frac{1}{2}$ |
| 2) P(4 or even) | 2) $\frac{1}{2}$ |
| 3) P(even or less than 4) | 3) $\frac{5}{6}$ |
| 4) P(odd or greater than 2) | 4) $\frac{5}{6}$ |
| 5) P(odd or prime) | 5) $\frac{2}{3}$ |
| 6) P(4 or less than 6) | 6) $\frac{5}{6}$ |

Marbles...

A jar contains four blue marbles and two red marbles. Suppose you choose a marble at random, and do not replace it. Then you choose a second marble. Find the probability of each.

- | | |
|---------------------------------|-----------|
| 1) Blue then red | 1) $4/15$ |
| 2) Red then a blue | 2) $4/15$ |
| 3) One is blue the other is red | 3) $8/15$ |
| 4) Both are red | 4) $1/15$ |

Is it Fair or not?

Fair or not?

1) There are 24 students in math class. The teacher wants to choose 4 students at random to come to the board and work a math problem. She writes each student's name on a slip of paper, places them in a hat, and chooses 4 without looking.

2) You and three friends want to choose which two of your group will shovel the snow in the driveway. You are each assigned a number from 1 to 4, and then spin a spinner to choose the first person. Then that person chooses the second person.

Using Random Numbers

- 1) The advisor of the good citizen's club wants to select 4 of its 25 members to raise and lower the flag each day this week. She assigns a two digit number from 01 to 25 to each student. What are the numbers that correspond to the members who will raise and lower the flag?
- 2) A coach wants to select 5 of his 16 players at random to help with a youth basketball camp this weekend. He assigns a two digit number from 01 to 16 to each player what are the numbers of the players who will help at camp?
- | |
|-------|
| 84496 |
| 18732 |
| 60330 |
| 19536 |
| 58380 |
| 52544 |
| 48712 |
| 01603 |
| 48862 |
| 18519 |
| 29834 |
| 90890 |
| 69751 |
| 20514 |